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Strength and Stability Analysis of Cassini Ovaloidal Heads of the Cylindrical Pressure Vessel with Geometrical Imperfections

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Abstract: Considering the wide application of cylindrical geometry in the construction of the body of aerial, marine and ground structures, the design of the heads connected to the body according to the placement conditions while maintaining the stress continuity at the connection point is of great importance. In this article, using the existing geometric relationships for special space shells, including Cassini oval shells of revolution, they are used as dished-end connected to the cylindrical pressure vessel and to investigate their strength and stability against external pressure as well as their buckling analysis. After extracting the relations related to the geometry of Cassini shells and establishing the boundary conditions for the connection of the shell to the cylinder, the stress analysis is performed based on the theory of membrane stress. Conversely, by creating the geometry of Cassini shells in Abaqus software and using scripting with Python, dimensionless stresses are extracted for shells with different shape parameters. Comparing the results of analytical and numerical solutions in this case shows the correctness of the analysis process. In the following, while extracting the critical load related to linear buckling, the effect of defects of different shapes (resulting from the superposition of the shapes of linear buckling modes) on the post-buckling behavior of Cassini heads has been investigated.
Keywords: Cassini ovaloidal heads; Strength and stability analysis; Linear and nonlinear buckling critical load; Geometrical imperfections.

1. Introduction

Researchers have discussed rotary shells in detail in many references. Timoshenko and Woinowsky-Krieger [1], Grigolûk and Kabanov [2], Ventsel and Krauthammer [3], Tovstik and Smirnov [4] can be mentioned as examples. Such shells are the basic components of thin-walled structures such as pressure vessels, liquid storage tanks, space carriers or submersible vehicles.

These types of structures are usually subjected to internal or external pressure, which in most cases is uniformly applied to the entire surface. For such loading, the most suitable shape is a shell with positive Gaussian curvature. In addition to spherical shells, which are an ideal solution for such loads, barreled shells have been the focus of many researchers. These are cylindrical shells with positive or negative meridian curvature.

Barreled shells have the advantages of cylindrical and spherical shells at the same time. A report on the optimization of fuel tanks used at the Marshall Space Flight Center states that "The end closure of the tank is usually elliptical, but may not be the best configuration for performance and cost; The head of the fuel tank and liquid oxygen tank are primarily designed for internal pressure; The Cassini head can be designed with the same volume as the elliptical head, in a shorter length and with less discontinuity at the edges to save the net weight of the device"; This is a good example of the semi-elliptical shape that is the best choice for fuel tanks [5].

The results of tests on several barreled shells used in a pressure body are presented by Blachut and Smith [6]. Theoretical and experimental analysis of the plastic buckling of the widened semi-elliptical dome at the poles under hydrostatic pressure with an applied approach in marine engineering was presented by Ross and his colleagues [7]. Smith and Blachut [8] presented numerical and experimental finite element analysis of prolate oval heads under external pressure, for several geometries with different thicknesses and boundary conditions. Blachut et al. have reported a series of numerical and experimental studies on the buckling performance of the hemispherical head [9], the dome under external pressure and also their buckling sensitivity.

If the input load reaches a critical value, it may be a disaster in practice. However, at this stage of the design process, it is possible to influence the behavior of a structure. One of the first works that mentioned this issue is the works of Reitinger and Ramm [10], Godoy [11], Bochenek [12] and Mróz and Piekarski [13]. In the above articles, the authors discuss optimization methods that incorporate the stability criterion. Other works that are dedicated to the stabilization of the post-buckling behavior of the shell were presented by Bochenek [14], Król et al. [15] and Krużelecki and Trybuła [16], in which the authors reach the goal by using preload forces or special support conditions. Another solution to obtain a stable post-buckling behavior is to make changes to the shape of Cassini's shell generator. Such an approach is presented for example by Jasion [17] and Singh [18]. A critical failure mode of ellipsoidal heads under internal pressure is investigated by Zheng et al. [19]. They developed a simple formula to calculate plastic collapse pressure of ellipsoidal heads. Tang et al. [20] numerically and experimentally analyzed the buckling of ellipsoidal pressure hulls with stepwise wall thicknesses. Theoretical and experimental study of barreled frustum [21] as well as the egg-shaped shells [22] and longan-shaped pressure hull [23] is accomplished by Zhang et al.

In this research, by using the existing geometric relationships for special space shells, including Cassini's elliptical rotary shells, they were used as heads connected to the cylindrical pressure body and to check their strength and stability against external pressure, as well as their buckling analysis. After extracting the relations related to the geometry of the Cassini shells and establishing the boundary conditions for connecting the shell to the cylinder, the stress analysis is performed based on the theory of membrane stress. On the other hand, by creating the geometry of Cassini shells in Abaqus software and using Python scripting, dimensionless stresses are extracted for shells with different shape parameters.

2. General geometry of Cassini dished head

As can be seen in Figure 1, in Cassini's quasi-elliptical shells, by choosing the relationships related to its flat type and considering different shape parameters, it is possible to investigate various types of plate heads connected to the cylinder body B.

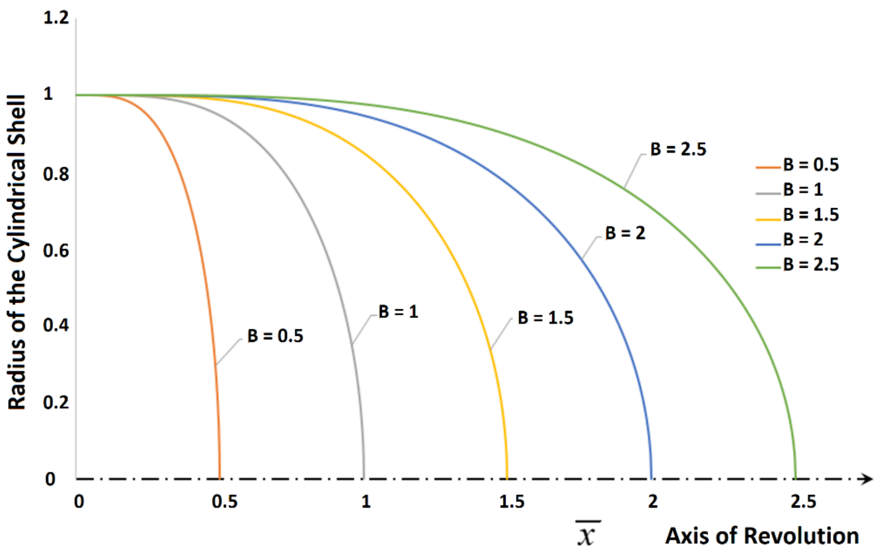

Figure 1. Cassini ovaloidal heads with different shape parameters

The Cassini ellipse is a plane curve of the second degree, which is described as a set of points that have the same interval multiplication output from two fixed points. Considering the desired shape of the head, only the flat ellipse (Figure 1) is considered. Such a choice ensures the continuity of the generative radius in the connection of the pressure vessel to the head. To change the relative depth of the head, the standard geometry of the Cassini ellipse is described according to Figure 2 [24].

In Figure 2, R_1 is the fundamental generative radius and R_2 is the peripheral radius of the Cassini shell. The necessary and sufficient conditions to remove the edge effects at the connection point of the head to the cylindrical body and also to ensure the continuity of the curvature of the Cassini shell at the desired point are:

$$k_1(0) = 0, \tag{1}$$

$$\left.\frac{dk_1}{dx}\right|_{x=0} = 0. \tag{2}$$

where k_1 is the curvature of the Cassini shell and is defined as the following relationship:

$$k_1 = \frac{1}{R_1} \tag{3}$$

It should be noted that the full expansion of equation (3) is not presented due to its cumbersome form.

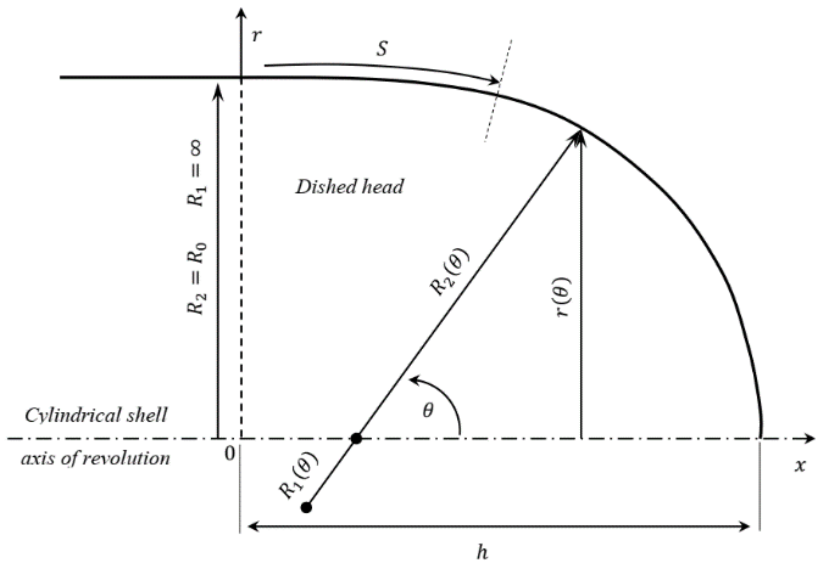

Figure 2. The generative sector of the Cassini head

In order to investigate the effect of the depth of the Cassini heads, the standard geometry of this type of heads is described as follows.

$$r(\overline{x}) = R_0 \overline{r}(\overline{x}) \tag{4}$$

$$\overline{r}(\overline{x}) = \sqrt{-1 - 3\frac{\overline{x}^2}{B^2} + 2\sqrt{1 + 3\frac{\overline{x}^2}{B^2}}} \tag{5}$$

Cassini heads, which can be extracted based on equation (5) for different values of *B* parameters, are shown in Figure 1.

The generative radius R1 and its dimensionless equivalent for the Cassini ellipse are obtained according to the following equation:

$$\boldsymbol{R_1 = R_0 \overline{R}_1} \tag{6}$$

$$\overline{R}_1 = -\frac{1}{3}B^2 \frac{\left[1 + \frac{9}{B^4}\frac{(2 - A_1(\overline{x}))^2}{A_1(\overline{x})^2 \overline{r}(\overline{x})^2}\overline{x}^2\right]^{\frac{3}{2}}}{\frac{2 - A_1(\overline{x})}{A_1(\overline{x})\overline{r}(\overline{x})} - \frac{3}{B^2}\frac{(2 - A_1(\overline{x}))^2 A_1(\overline{x}) + 8\overline{r}(\overline{x})^2}{A_1(\overline{x})^3 \overline{r}(\overline{x})^3}\overline{x}^2} \tag{7}$$

Where:

$$A_1(\overline{x}) = 2\sqrt{1 + 3\frac{\overline{x}^2}{B^2}} \tag{8}$$

Also, the circumference radius of the Cassini ellipse is obtained by the following relationship.

$$R_2 = R_0 \overline{r}(\overline{x})\sqrt{1 + \frac{9}{B^4}\frac{(2 - A_1(\overline{x}))^2}{A_1(\overline{x})^2 \overline{r}(\overline{x})^2}\overline{x}} \tag{9}$$

The length of the modified Cassini ellipse generative curve can be obtained according to relations 10 and 11.

$$S = R_0 \overline{S} \tag{10}$$

$$\overline{S} = \int_0^B \sqrt{1 + \frac{9}{B^4} \frac{(2 - A_1(\overline{x}))^2}{A_1(\overline{x}) r(\overline{x})^2} \overline{x}^2} d\overline{x} \tag{11}$$

3. Strength analysis

In strength analysis of shells under external or internal pressure, efforts should be focused on stress distribution. Edge effects or stress concentration caused by curvature discontinuity should be avoided. This is possible if the generative curve is continuous throughout the generative curve of the shell. This continuity ensures uniform stress distribution in the shell.

3.1 Membrane tension theory

Assuming that the thickness of the shell is insignificant compared to its smallest radius of curvature, it can be assumed that the theory of membrane stress can be used for such a shell. The internal forces of the shell under the external pressure load *p* can be defined as follows:

$$N_1 = \frac{1}{2} p R_2 \tag{12}$$

$$N_2 = \frac{1}{2} p R_2 \left(2 - \frac{R_2}{R_1}\right) \tag{13}$$

where, *p*: Uniform external pressure; R_1, R_2: Respectively, generative and circumference radius.

The stress components in the main directions are as follows:

$$\sigma_1 = \frac{N_1}{t_s}, \qquad \sigma_2 = \frac{N_2}{t_s}, \tag{14}$$

where t_s is the thickness of the Cassini shell. The equivalent von Mises stress for the desired shell can be expressed in terms of the main stress components in the form of equation 15:

$$\sigma_{eq} = \sqrt{\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2} \tag{15}$$

According to the above relations, for a cylindrical shell, it can be written as:

$$\sigma_{eq0} = \frac{\sqrt{3}}{2} \frac{p}{t_s} R_0 \tag{16}$$

where R_0 is the radius of the cylindrical shell.

In order to compare the amount of equivalent stress in different parts of the Cassini shell more easily, the following dimensionless equivalent stress is introduced.

$$\overline{\sigma} = \frac{\sigma_{eq\,h}}{\sigma_{eq0}} \tag{17}$$

where $\sigma_{eq\,h}$ is the equivalent von Mises stress of the Cassini head.

4. Nonlinear theory in the analysis of post-buckling behavior of structures

The concept of equilibrium path plays a central role in the nonlinear analysis of structures. One of the important tools for this equilibrium path is the system response diagram. The response diagram that is most used in nonlinear analysis is the transformation diagram.

In the analysis of the nonlinear behavior of the structure, certain points of the equilibrium path, such as limit points and bifurcation points, are particularly important. At limit points, the tangent to the equilibrium path is horizontal and two or more equilibrium paths pass through the bifurcation points. It is necessary to remember that in these critical points, the relationship between force and displacement is not unique and the structure is uncontrollable. These points are essential from the point of view of engineering because they are the points that the designer often avoids. This article intends to determine the equilibrium path of the structure and its critical points by using Riks analysis of Abaqus finite element software.

The Newton-Raphson iteration method is a common method for solving nonlinear equations governing the behavior of structures. In this method, the linear search process is used to increase the speed of convergence. This method is not able to pass the limit points in the equilibrium path [25]. The powerful arc-length method, in which the iterative solution is constrained and follows the deterministic path of equilibrium, is capable of solving limit point problems by combining load and displacement growth. This method has been well used in Riks algorithm in Abaqus software in analyzing the buckling behavior of shells. An overview of the intended analysis is explained below. Equation 18 is considered as a condition for establishing equilibrium and a basis for deriving the equilibrium path of the structure.

$$R(x,\lambda) = T(x) - \lambda\bar{F} = 0 \tag{18}$$

In equation (18), *F* represents the equivalent nodal external load distribution, *T* represents the internal forces and *R* is the residual. Now, the value of *λ* changes during the repetition of the Newton-Raphson process by imposing an additional conditional relation. Based on proportional loading, the *i*-th load growth is obtained by the growth in the value of *λ*, and this value will differ (-1*i*) from the value obtained at the end of the previous load growth:

$$\Delta F_1 = \Delta\lambda\bar{F}, \quad \Delta\lambda = \lambda - \lambda_{i-1} \tag{19}$$

Similarly, the total change in the position (displacement) of the structure under the effect of load growth is represented by Δx and as follows:

$$\Delta x = x_i - x_{i-1} \tag{20}$$

5. Results

In the first step of this article, the stress and strength analysis of Cassini heads has been investigated. In this regard, using equations 12 to 16, the dimensionless equivalent stress of Cassini heads with different depth parameters is calculated.

As mentioned earlier, in this article, scripting has been used to produce the exact geometry of Cassini's heads in Abaqus software. In this regard, the equations related to the shell geometry (equations 4 and 5) were added to the Python code prepared for the stress analysis of the Cassini heads, and the results related to stress analysis and linear buckling were extracted by creating a repetition loop.

In order to check the accuracy of the calculations and to ensure the independence of the results from the mesh, the Cassini head with the shape parameter B=0.5 and the sweep mesh algorithm have been investigated. The convergence behavior of the results with increasing the density of elements is shown in Figure 3. The following relationship is used to calculate the relative error.

$$\text{Relative Error (\%)}=\left\|\frac{\text{present solution-exact solution}}{\text{exact solution}}\right\|\times 100. \tag{21}$$

Using equation 3, the changes in the curvature of the generator (K_1) related to the shell of the Cassini heads for different values of the shape parameter *B* compared to the length of the curve are shown in Figure 4. In the following and in a comparative study, the equivalent stress extracted from the analysis with membrane theory and the stress extracted from the finite element software for Cassini shells with different shape parameters is presented in Figure 5. Observations indicate that there is a good agreement between the results of analytical and numerical solutions.

In the next step of this article, the stability of the heads made of Cassini shells has been investigated. Due to the complexity of the geometry of these kinds of shells, it is practically impossible to extract the analytical relations of the closed form of buckling and particularly to satisfy the boundary conditions. Therefore, in this article, the Buckle solver of Abaqus software is used to extract the critical load of linear buckling of Cassini shells. It is necessary to explain that the validation of the desired finite element code for the hemispherical head for which analytical relations are available was done by this research team. In Figure 6, the values related to the critical buckling load of Cassini head shells with different shape parameters are presented. The desired values are reported for heads with radius R_0=1000 mm, thickness=8 mm and modulus of elasticity E=205 GPa. As it is clear from the diagram, the highest value of the critical load is for the Cassini head with a shape parameter close to one (B=1). The shape of the mode corresponding to the desired critical loads for different heads can be seen in Figure 7. As shown in this figure, the shape of the first mode in Cassini heads with different shape parameters is formed at the tip of the head.

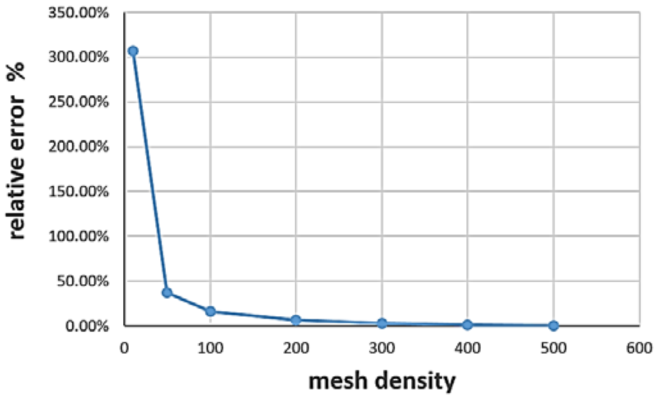

Figure 3. Mesh convergence for Cassini head (B = 0.5)

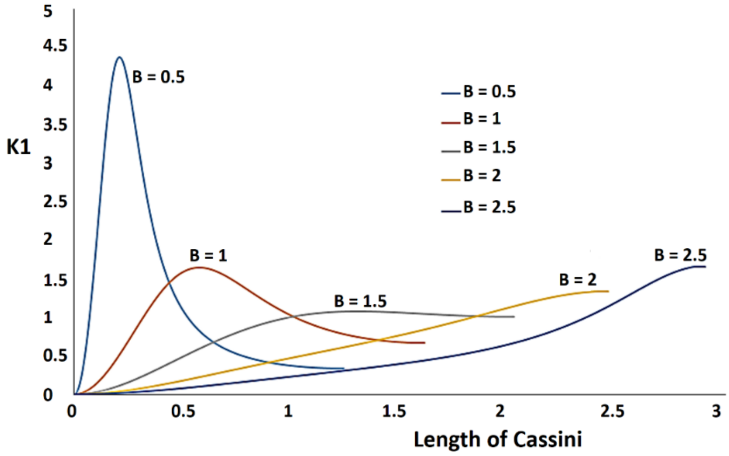

Figure 4. Generative curvature in the shells of the Cassini head

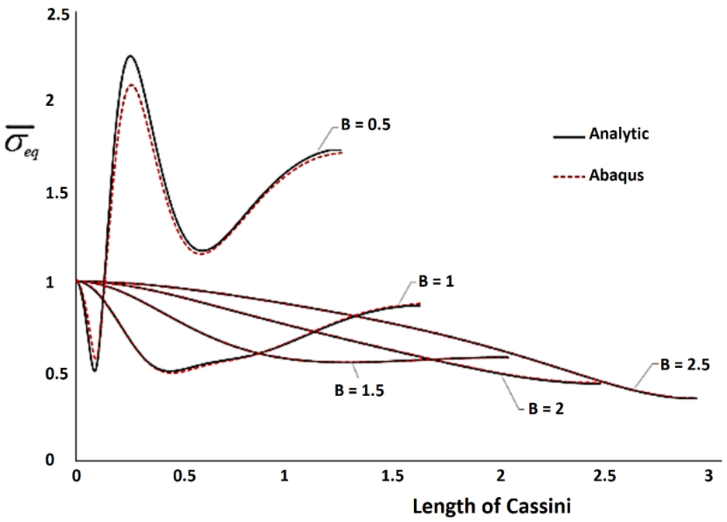

Figure 5. Dimensionless equivalent stress in the shells of the Cassini head

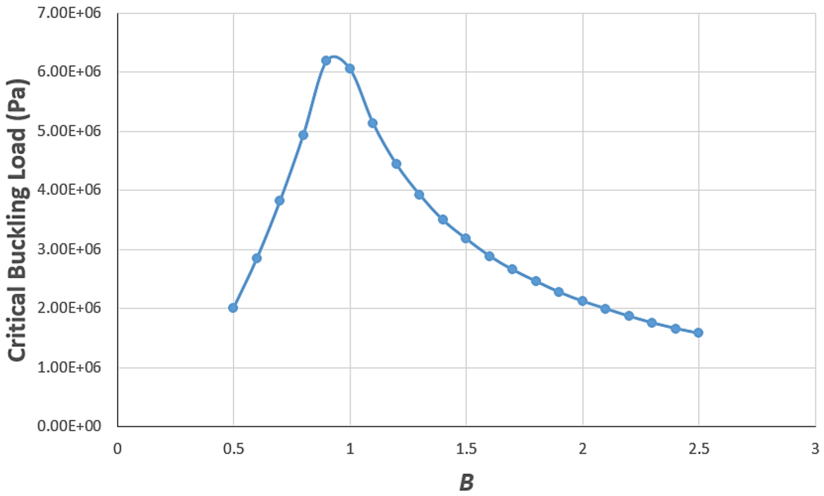

Figure 6. Critical load of linear buckling of Cassini heads

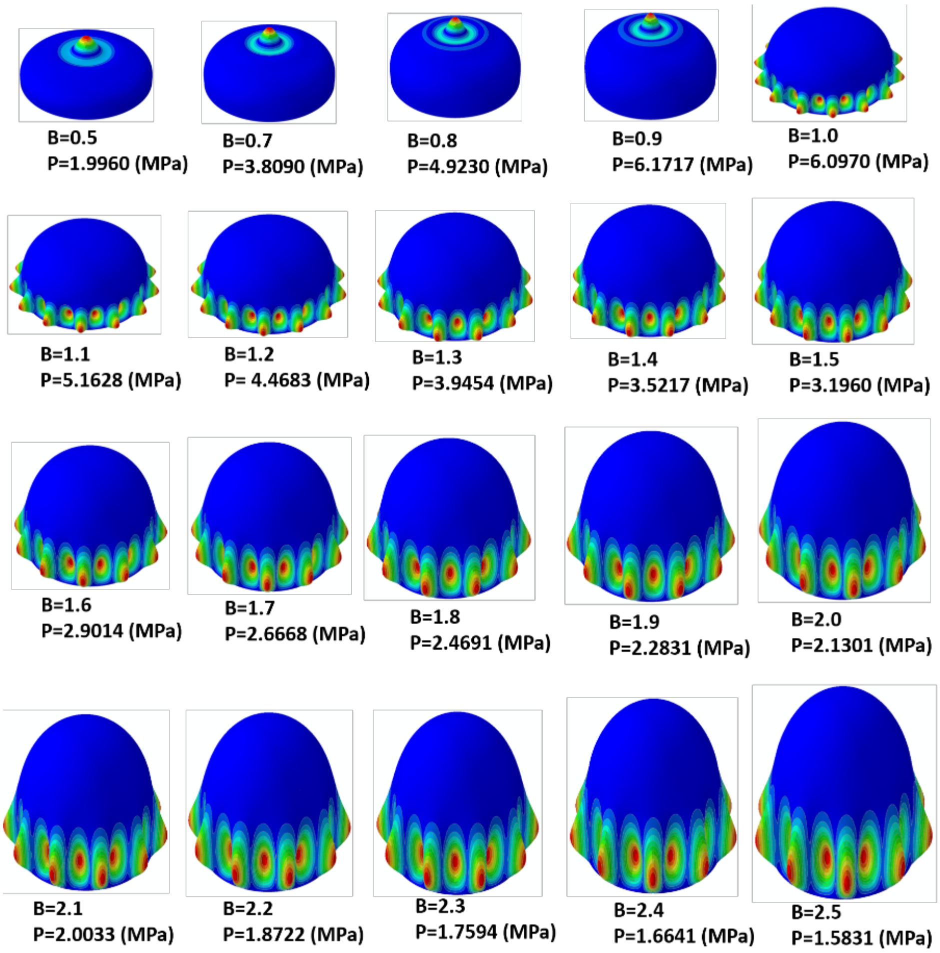

Figure 7. First mode shape of Cassini shells with different shape parameters

After performing the linear buckling analysis of Cassini shells and in the continuation of the activities of this article, the post-buckling behavior of these shells will be investigated under external hydrostatic pressure. Due to the high sensitivity of the buckling behavior of shells with positive Gaussian curvature to geometric shape defects, their effect on Cassini heads has been investigated in this article. The shape defects are applied as a linear combination of the shapes of the linear modes and using the "imperfection" keyword in the "inp" file of Abaqus on the main model. The purpose of this analysis is to extract the equilibrium path of the structure and determine the critical load at the limit points located on the equilibrium path. The results related to the post-buckling analysis of the Cassini head with shape parameter *B*=1.5, thickness *t*=8 mm and elastic-perfect plastic behavior of the material and defects of different shapes are shown in Figure 8. In this case, the load on the shell is considered equivalent to the critical load of linear buckling. Naturally, with the increase in the amount of shape defects in the structure, we will see a decrease in the nonlinear critical load of the structure. Subsequently, the influence of the thickness on the behavior of the structure in the process of nonlinear buckling and under geometric shape defects has been investigated and its results have been presented in the diagram of Figure 9.

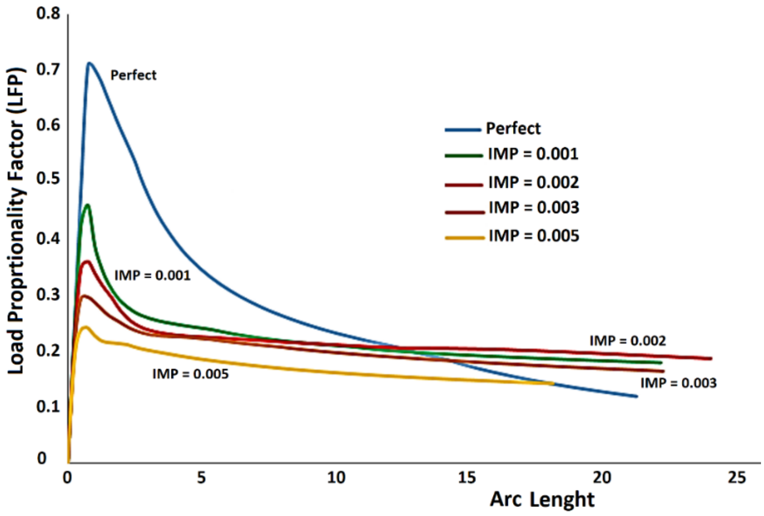

Figure 8. LPF diagram of Cassini's head with different geometric shape defects

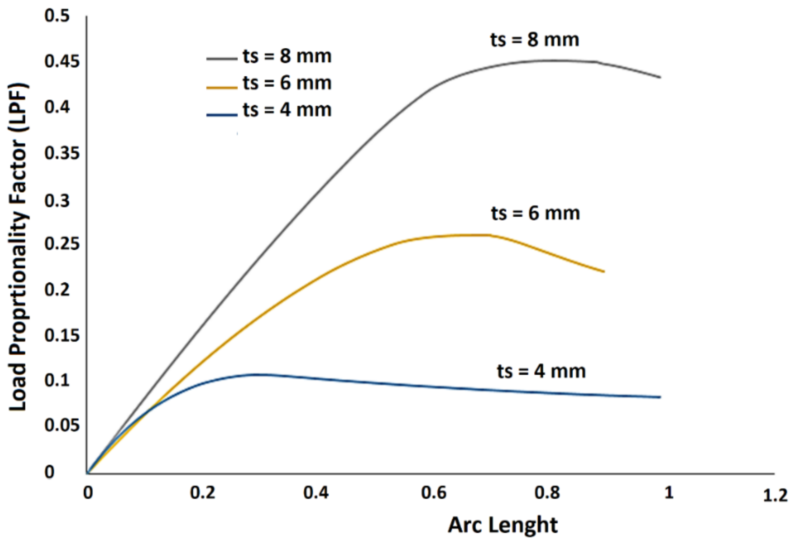

Figure 9. Effect of thickness on post-buckling behavior of Cassini head (imp=0.001)

6. Conclusion and summary

In this article, Cassini oval shells of revolution are introduced as dished-end attached to the cylindrical pressure vessels, and their strength and buckling behavior have been carefully investigated. The function of geometric changes of these shells has been extracted according to the requirements of connection to the cylindrical body, including the elimination of edge effects and by applying constraints on the equations of the Cassini shell. Membrane stress theory is the basis of stress analysis for these shells of revolution and the relevant equations have been analyzed through coding in MATLAB software for different shape parameters. In the following, the scripting capability in the Python platform is used in Abaqus finite element software and the equations related to the geometry of Cassini shells are modeled in the software. In this case, the stress analysis was performed on a 90-degree section of the rotary shell and the results were compared with the analytical solution, and a good match between the results was observed. In the following, using finite element software, the critical buckling load of Cassini heads and the shape of the corresponding modes have been extracted and the effect of different shape parameters on it has been investigated. After that, the effect of geometric shape defects and shell thickness on the post-buckling behavior of Cassini heads has been studied and the results have been presented. The highest value of the critical load is for the Cassini head with a shape parameter close to one (B=1).

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