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Effect of Nonlocal on Poro-thermoelastic Solid with Dependent Properties on Refrence Temperature via the Three-phase-lag Model

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Abstract: In this work, a novel nonlocal model on a poro-thermoelastic solid with temperature-dependent properties is presented. To solve this problem, the thermo-elasticity theory with three-phase-lag model (3PHL) is proposed. The modulus of the elasticity is given as a linear function of the reference temperature. The analytical expressions of the displacement components, the stresses, and the temperature are obtained by normal mode analysis. The main physical fields are displayed graphically and theoretically discussed under the influence of the nonlocal parameter and temperature-dependent properties.
Keywords: Temperature dependent properties; Nonlocal parameter; Porous solid; Three-phase-lag model; Thermoelasticity.

1. Introduction

The nonlocal elasticity theory was displayed to study many applications in nano-mechanics including lattice dispersion of elastic waves, waves propagation in composites materials, dislocation mechanics, fracture mechanics, etc. The theory of nonlocal continuum mechanics was proposed by Eringen [1]. Altan [2] studied the uniqueness of the solutions of a class of initial-boundary value problems in linear, isotropic, homo-geneous, nonlocal elasticity. Povstenko [3] discussed the nonlocal theory of elasticity and its applications to the description of defects in solid bodies. Recently, Lim et al. [4] constructed a higher order nonlocal elastic and strain gradient theory by combining the procedures of nonlocal and gradient elasticity. The nonlocal thermodynamics and the axiom of objectivity a set of constitutive equations is developed for the nonlocal thermo-elastic solids by Eringen [5]. Some theorems in generalized nonlocal thermoelasticity were discussed by Dhaliwal and Wang [6]. Das et al. [7] introduced the Green and Naghdi model II of thermoelasticity and the Eringen’s nonlocal elasticity model to study the propagation of harmonic plane waves in a nonlocal thermoelastic medium. Yu et al. [8] discussed the nonlocal thermo-elasticity based on nonlocal heat conduction and nonlocal elasticity. Zenkour [9] introduced the nonlocal thermoelasticity theory without energy dissipation for nano-machined beam resonators subjected to various boundary conditions. Sarkar et al. [10] displayed the effect of the laser pulse on transient waves in a non-local thermoelastic medium under Green-Naghdi theory. Luo et al. [11] introduced the nonlocal thermoelastic model to predict the thermoelastic behavior of nanostructures under extreme environments. Abbas et al. [12] displayed the analytical solutions for the nonlocal thermoelastic problem using Laplace transforms and the eigenvalue method. Lata and Singh [13] studied of the axisymmetric deformations in a two-dimensional non-local homogeneous isotropic thermoelastic solid without energy dissipation.

The thermal stress in a material with temperature-dependent properties was studied extensively by Noda [14]. Material properties such as the modulus of elasticity and the thermal conductivity vary with temperature. When the temperature variation from the initial stress is not strongly varying, the properties of materials are constants. In the refractory industries, the structural components are exposed to a high temperature change. In this case, neglecting the temperature dependence will result in errors in material properties as Jin and Batra [15]. Many material properties, such as Young’s modulus, coefficient of thermal expansion, and yield stress, can have a significant dependence on temperature, some studies in temperature-dependent are due to Othman et al. [16-21]. The generalized thermoelasticity proposed by Green and Lindsay, the dynamic response of generalized thermoelastic problems with temperature-dependent material properties is investigated by He and Shi [22]. Barak and Dhankhar [23] concerned with the solution of a problem on thermoelastic interactions in a functionally graded (non-homogeneous), fiber-reinforced, transversely isotropic half-space with temperature-dependent properties under the application of an inclined load in the context of Green-Naghdi theory of type III.

The aim of this present study is to determine the distributions of the displacement components, the stresses and the temperature in a nonlocal poro-thermoelastic solid with temperature dependent properties. A novel nonlocal model study is illustrated in the context the three-phase lag model. The non-dimensional coupled governing equations are solved using the normal mode analysis. The numerical results are given and presented graphically to show the effect of the nonlocal parameter and temperature-dependent properties on a poro-thermoelastic solid.

2. The discussion of the problem and the basic equations

The problem of a rotating nonlocal porous thermoelastic solid with temperature dependent properties. We are interested in xy-plane and our dynamic displacement is given as $\boldsymbol{u} = (u, v, 0)$, $w = 0$, $\frac{\partial}{\partial z} = 0.$

The constitutive equations as Hetnarski and Eslami [24], Eringen et al. [1, 5, 25] and Abd-Elaziz et al. [26]:

$$(1-\varepsilon^2\nabla^2)\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} + b\varphi\delta_{ij} - \gamma\theta\delta_{ij}, \tag{1}$$

where, $\varepsilon = a_0 e_0$ is the elastic nonlocal parameter having a dimension of length, a_0, e_0 respectively, are an internal characteristic length and a material constant, $\sigma_{ij}$are the components of stress, $e_{ij}$are the components of strain, e_{kk} is the dilatation, λ, μ are elastic constants, α_t is the thermal expansion coefficient, $\varphi$is the change in volume fraction field of voids, $\theta = T - T_0$, where $T$is the temperature above the reference temperature T_0, and $\delta_{ij}$is the Kronecker's delta.

The equation of motion

$$\rho\ddot{u}_i = \sigma_{ji,j}, \tag{2}$$

$$\beta\varphi_{,ii} - be - \alpha_1\varphi - \alpha_2\varphi_{,t} + \alpha_3\theta = \rho\alpha_4(1-\varepsilon^2\nabla^2)\varphi_{,tt}, \tag{3}$$

where $\beta, b, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the material constants due to the presence of voids Said et al. [27].

The heat conduction equation as Choudhuri [28]

$$K(1+\tau_\theta\frac{\partial}{\partial t})\nabla^2\theta_{,t} + K^*(1+\tau_\nu\frac{\partial}{\partial t})\nabla^2\theta = (1+\tau_q\frac{\partial}{\partial t}+\frac{1}{2}\tau_q^2\frac{\partial^2}{\partial t^2})(\rho C_E\theta_{,tt} + \gamma T_0 e_{,tt} + \alpha_3 T_0\varphi_{,tt}), \tag{4}$$

where K^* is the coefficient of thermal conductivity, K is the additional material constant,

C_E is the specific heat at constant strain, τ_ν is the phase-lag of thermal displacement gradient, τ_θ is the phase-lag of temperature gradient and τ_q is the phase-lag of heat flux.

We may assume that as Ezzat et al. [29]:

$$\mu = \mu_1 f(T),\quad \lambda = \lambda_1 f(T),\quad b = b_1 f(T),\quad \alpha_1 = \alpha_{11} f(T),\quad \alpha_2 = \alpha_{21} f(T),\quad \alpha_3 = \alpha_{31} f(T),\quad \alpha_4 = \alpha_{41} f(T),$$
$$\beta = \beta_1 f(T),\quad \gamma = \gamma_1 f(T), \tag{5}$$

where $\mu_1, \lambda_1, b_1, \alpha_{11}, \alpha_{21}, \alpha_{31}, \alpha_{41}, \gamma_1, \beta_1$ are constants of material, $f(T) = (1 - \alpha^* T_0)$ and α^* is an empirical material constant. In the case of the temperature independent modulus of elasticity, and thermal conductivity $\alpha^* = 0$.

For convenience, we introduce the non-dimension variables as:

$$(x', y', \varepsilon', u', v') = \frac{1}{l_0}(x, y, \varepsilon, u, v),\ (t', \tau'_q, \tau'_\theta, \tau'_\nu) = \frac{c_0}{l_0}(t, \tau_q, \tau_\theta, \tau_\nu),\ \theta' = \frac{\gamma\theta}{(\lambda+2\mu)},\ \sigma'_{ij} = \frac{\sigma_{ij}}{\mu},$$
$$\varphi' = \varphi,\quad l_0 = \sqrt{\frac{K^*}{\rho C_E T_0}},\quad c_0 = \sqrt{\frac{\lambda+2\mu}{\rho}}. \tag{6}$$

Using the above non-dimensional variables defined in Eq. (6) and Eqs. (1) in Eqs. (2)- (4), we get

$$(1-\varepsilon^2\nabla^2)\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + B_1\frac{\partial^2 v}{\partial x\,\partial y} + B_2\frac{\partial^2 u}{\partial y^2} - \frac{\partial\theta}{\partial x} + B_3\frac{\partial\varphi}{\partial x},, \tag{7}$$

$$(1-\varepsilon^2\nabla^2)\frac{\partial^2 v}{\partial t^2} = B_2\frac{\partial^2 v}{\partial x^2} + B_1\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial^2 v}{\partial y^2} - \frac{\partial\theta}{\partial y} + B_3\frac{\partial\varphi}{\partial y}, \tag{8}$$

$$B_4(1+\tau_\theta \frac{\partial}{\partial t})\nabla^2\theta_{,t} + (1+\tau_v \frac{\partial}{\partial t})\nabla^2\theta = (1+\tau_q \frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2 \frac{\partial^2}{\partial t^2})(B_5\,\theta_{,tt} + B_6 e_{,tt} + B_7\ \varphi_{,tt}), \tag{9}$$

$$\varphi_{,ii} - B_8 e - B_9\varphi - B_{10}\varphi_{,t} + B_{11}\theta = B_{12}(1-\varepsilon^2\nabla^2)\varphi_{,tt}, . \tag{10}$$

where,

$$B_1 = \frac{\lambda+\mu}{\rho c_0^2},\quad B_2 = \frac{\mu}{\rho c_0^2},\quad B_3 = \frac{b}{\rho c_0^2},\ B_4 = \frac{K c_0}{K^* l_0},\quad B5 = \frac{\rho C_E c_0^2}{K^*},\quad B_6 = \frac{\gamma^2 T_0\ c_0^2}{K^*(\lambda+2\mu)},$$
$$B_7 = \frac{\alpha_3 B_6}{\gamma},\quad B_8 = \frac{b l_0^2}{\beta},\quad B_9 = \frac{\alpha_1 l_0^2}{\beta},\quad B_{10} = \frac{\alpha_2 l_0 c_0}{\beta},\quad B_{11} = \frac{\alpha_3 l_0^2(\lambda+2\mu)}{\gamma\beta},\ B_{12} = \frac{\rho\alpha_4 c_0^2}{\beta}.$$

3. The analytical solution of the problem

The solution of the considered physical variable can be decomposed in terms of normal mode analysis as:

$$[u, v, \theta, \varphi, \sigma_{ij}](x, y, t) = [\bar{u}, \bar{v}, \bar{\theta}, \bar{\varphi}, \bar{\sigma}_{ij}](x)\exp(\mathrm{i}ay - mt). \tag{11}$$

where $\bar{u}(x)$, etc. is the amplitude of the function $u(x,y,t)$ etc., i is the imaginary unit, m (complex) is the time constant and a is the wave number in the y −direction.

Introducing Eq. (11) in Eqs. (7)−(10), we get

$$(M_1\mathrm{D}^2 - M_2)\,\bar{u} - \mathrm{i}aB_1\mathrm{D}\,\bar{v} - B_3\mathrm{D}\bar{\varphi} + \mathrm{D}\bar{\theta} = 0, \tag{12}$$

$$-\mathrm{i}a B_1\mathrm{D}\bar{u} + (M_3\mathrm{D}^2 + M_4)\bar{v} - \mathrm{i}aB_3\bar{\varphi} + \mathrm{i}a\,\bar{\theta} = 0, \tag{13}$$

$$B_8\mathrm{D}\bar{u} + \mathrm{i}aB_8\bar{v} - (M_5\mathrm{D}^2 - M_6)\,\bar{\varphi} - B_{11}\bar{\theta} = 0, \tag{14}$$

$$M_7\,\mathrm{D}\bar{u} + \mathrm{i}aM_7\bar{v} + M_8\bar{\varphi} - (M_9\mathrm{D}^2 - M_{10})\bar{\theta} = 0, \tag{15}$$

where,

$$M_1 = -(\varepsilon^2 m^2 + 1),\qquad M_2 = (1+\varepsilon^2 a^2)(-m^2) - B_2 a^2,\qquad M_3 = -\varepsilon^2 m^2 - B_2,\qquad M_4 = (1+\varepsilon^2 a^2)m^2 + a^2,$$
$$M_5 = 1 + \varepsilon^2 m^2 B_{12},\qquad M_6 = m^2 B_{12}(1+a^2\varepsilon^2) + a^2 + B_9 - m B_{10},\qquad M_7 = m^2 B_6(1 - m\tau_q + 0.5m^2\tau_q^2),$$
$$M_8 = \frac{B_7 M_7}{B_6},\qquad M_9 = mB_4(m\tau_\theta - 1) + 1 - m\tau_v,\quad M_{10} = a^2 M_9 + m^2 B_5(1 - m\tau_q + 0.5m^2\tau_q^2).$$

Eliminating $\bar{u}(x), \bar{v}(x)$, and $\bar{\theta}(x)$ between Equations (12) – (15), the following ordinary differential equation can be obtained:

$$(\mathrm{D}^8 - E_1\mathrm{D}^6 + E_2\mathrm{D}^4 - E_3\mathrm{D}^2 + E_4)\bar{\phi}(x) = 0. \tag{16}$$

where, $E_1 = \frac{L_1}{L_5}, E_2 = \frac{L_2}{L_5}, E_3 = \frac{L_3}{L_5}, E_4 = \frac{L_4}{L_5}$,

$$L_1 = M_5 M_{10} + M_6 M_9 - M_3 M_5 M_7 - B_1^2 M_5 M_9 a^2 + B_3 B_8 M_3 M_9 + M_1 M_3 M_5 M_{10}$$
$$+ M_1 M_3 M_6 M_9 - M_1 M_4 M_5 M_9 + M_2 M_3 M_5 M_9 + 2M_5 M_9,$$

$$L_2 = B_{11} M_8 + M_5 M_9 + M_6 M_{10} + B_8 M_3 M_8 - M_3 M_6 M_7 + M_4 M_5 M_7 - B_1^2 M_5 M_{10} a^2$$
$$- B_1^2 M_8 M_{11} a^2 + B_3 B_8 M_5 M_{12} - B_3 B_8 M_4 M_9 + B_3 B_{11} M_3 M_7 + B_{11} M_1 M_3 M_8 + M_1 M_3 M_6 M_{10}$$
$$- M_1 M_4 M_5 M_{10} - M_1 M_4 M_6 M_9 + M_2 M_3 M_5 M_{10} + M_2 M_3 M_6 M_9 - M_2 M_4 M_5 M_9 + 2M_5 M_{10}$$
$$+ 2M_6 M_9 - 2B_1 M_5 M_7 a^2 - M_1 M_5 M_7 a^2 + 2B_1 B_3 B_8 M_9 a^2 + B_3 B_8 M_1 M_9 a^2,$$

$$L_3 = M_5 M_{10} + M_6 M_9 - B_8 M_4 M_8 + M_4 M_6 M_7 - B_1^2 M_6 M_{10} a^2 - B_3 B_8 M_4 M_{10} - B_3 B_{11} M_4 M_7$$
$$- B_{11} M_1 M_4 M_8 + B_{11} M_2 M_3 M_8 + 2B_{11} M_8 - M_1 M_4 M_6 M_{10} + M_2 M_3 M_6 M_{10} - M_2 M_4 M_5 M_{10}$$

$$-M_2M_4M_6M_9 + 2M_6M_{10} + 2B_1B_8M_8a^2 - 2B_1M_6M_7a^2 + B_8M_1M_8a^2 - M_1M_6M_7a^2$$
$$-M_2M_5M_7a^2 - B_1^2B_{11}M_8a^2 + 2B_1B_3B_8M_{10}a^2 + 2B_1B_3B_{11}M_7a^2 + B_3B_8M_1M_{10}a^2$$
$$+B_3B_8M_2M_9a^2 + B_3B_{11}M_1M_7a^2,$$
$$L_4 = -M_2M_6M_7a^2 + M_6M_{10} + B_{11}M_8 - B_{11}M_2M_4M_8 - M_2M_4M_6M_{10} + B_8M_2M_8a^2$$
$$+B_3B_8M_2M_{10}a^2 + B_3B_{11}M_2M_7a^2,$$
$$L_5 = M_5M_9 + M_1M_3M_5M_9.$$

The solution of Eq. (16), which is bounded as $x \to \infty$, is given by

$$\bar{\varphi}(x) = \sum_{j=1}^{4} C_j \exp(-k_j x). \tag{17}$$

Similarly,

$$\bar{\theta}(x) = \sum_{j=1}^{4} L_{1j} C_j \exp(-k_j x), \tag{18}$$
$$\bar{u}(x) = \sum_{j=1}^{4} L_{2j} C_j \exp(-k_j x), \tag{19}$$
$$\bar{v}(x) = \sum_{j=1}^{4} L_{3j} C_j \exp(-k_j x). \tag{20}$$

Using the above results, we get

$$\bar{\sigma}_{xx}(x) = \sum_{j=1}^{4} L_{4j} C_j \exp(-k_j x), \tag{21}$$
$$\bar{\sigma}_{xy}(x) = \sum_{j=1}^{4} L_{5j} C_j \exp(-k_j x), \tag{22}$$

where $k_j^2 (j = 1,2,3,4)$ are the roots of the characteristic equation: $\mathrm{k}^8 - E_1\mathrm{k}^6 + E_2\,\mathrm{k}^4 - E_3\mathrm{k}^2 + E_4 = 0.$

$$L_{1j} = \frac{M_6M_7 - M_5M_7k_j^2 - B_8M_8}{B_8M_{10} - B_8M_9k_j^2 + B_{11}M_7},$$
$$L_{2j} = \frac{L_{1j}(a^2B_8 - B_{11}M_3k_j^2 - B_{11}M_4) - (a^2B_3B_8 + M_3M_5k_j^4 - M_3M_6k_j^2 + M_4M_5k_j^2 - M_4M_6)}{-a^2B_1B_8k_j + B_8M_3k_j^3 + B_8M_4k_j},$$
$$L_{3j} = \frac{B_8k_jL_{2j} + B_{11}L_{1j} + M_5k_j^2 - M_6}{\mathrm{i}aB_8}, \quad L_{4j} = \frac{\mathrm{i}a\lambda L_{3j} - (\lambda+2\mu)(k_jL_{2j} + L_{1j}) + b}{\mu(1+\varepsilon^2a^2 - \varepsilon^2k_j^2)}, \quad L_{5j} = \frac{\mathrm{i}a\mu L_{2j} - \mu k_j L_{3j}}{\mu(1+\varepsilon^2a^2 - \varepsilon^2k_j^2)}.$$

4. The boundary conditions

The mechanical and thermal boundary conditions at the thermally stress-free surface at $x = 0$ are
a) Thermal boundary condition: in which the surface of the half-space is subjected to

$$\theta = 0. \tag{23}$$

b) Mechanical boundary condition: in which surface of the half-space is subjected to

$$\sigma_{xx} = -f_1(y,t). \tag{24}$$

c) Mechanical boundary condition: in which surface of the half-space is subjected to

$$\sigma_{xy} = 0. \tag{25}$$

d) Condition on the change in volume fraction field

$$\phi = g(y,t). \tag{26}$$

where $f(y,t) = f_1\, exp(\, iay - mt)$, $g(y,t) = f_2\, exp(\, iay - mt)$ and f_1, f_2 are constants. Substituting the expressions of the variables considered into the above boundary conditions, we can obtain the following equations satisfied by the parameters:

$$\sum_{j=1}^{4} L_{1j} C_j = 0, \qquad \sum_{j=1}^{4} L_{4j} C_j = -f_1, \qquad \sum_{j=1}^{4} L_{5j} C_j = 0, \quad \sum_{j=1}^{4} C_j = f_2. \tag{27}$$

Applying the inverse of matrix method, we have

$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{41} & L_{42} & L_{43} & L_{44} \\ L_{51} & L_{52} & L_{53} & L_{54} \\ 1 & 1 & 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ -f_1 \\ 0 \\ f_2 \end{pmatrix}. \tag{28}$$

5. Numerical calculations and discussion

To illustrate the theoretical results obtained in the previous section, to compare them in the context of different thermoelastic theories, and to study the influence of position- and temperature-dependent properties on wave propagation in porous thermoelastic media, we now present some values of the physical constants results, as in Othman et al. [21]

$$\lambda_1 = 3.9\,\mathrm{x}\,10^{10}\ \mathrm{N.m^{-2}}, \quad \mu_1 = 7.78\,\mathrm{x}\,10^{10}\ \mathrm{N.m^{-2}}, \quad \rho = 8954\ \mathrm{kg.m^{-3}}, \quad C_E = 383\,\mathrm{J.kg^{-1}.K^{-1}}, \quad \alpha_t = 3.78\ \mathrm{x}10^{-4}\ \mathrm{K^{-1}},$$

$$f_1 = -0.005, \quad \tau_q = 9\mathrm{x}10^{-7}\mathrm{s}, \quad \tau_\nu = 6\mathrm{x}10^{-7}\mathrm{s}, \quad \tau_\theta = 7\mathrm{x}10^{-7}\mathrm{s}, \quad K^* = 386\,\mathrm{w.m^{-1}.K^{-1}.s^{-1}}, \quad b_1 = 1.6\mathrm{x}10^{10}\mathrm{N.m^{-2}}, \quad \alpha_{11} = 1.47\mathrm{x}10^{10}\mathrm{N.m^{-2}},$$

$$\alpha_{21} = 7.78\mathrm{x}10^{-10}\mathrm{N.m^{-2}}, \ m = m_0 + i\xi, \quad m_0 = 0.6, \ \xi = 0.08, \ a = 0.5, \ T_0 = 293\mathrm{K}, \ K = 386\,\mathrm{w.m^{-1}.K^{-1}}, \ f_2 = 10^{-4},$$

$$\alpha_{31} = 2\,\mathrm{x}10^{10}\mathrm{N.m^{-2}}, \ \alpha_{41} = 1.753\mathrm{x}10^{-11}\mathrm{N.m^{-2}}, \ \beta_1 = 2\,\mathrm{x}10^{10}\mathrm{N.m^{-2}}, \ y = 20$$

The two cases were compared using a three-phase lag model.
(i) Linear temperature coefficient with three different values ($\alpha^* = 0.0005,\ 0.0008,\ 0.002$).
(ii) non-local parameters with three different values ($\varepsilon = 0, 0.05, 0.5$).

Case 1: In Figure 1-5, calculate the distribution of displacement component v, thermal temperature θ, volume change field φ and stress components $\sigma_{xx},\ \sigma_{xy}$at time $t = 0.5\ s$. The results in the figures are dimensionless. Figure 1 shows the variation of displacement v versus x. The offset starts with a positive value, and even though the size of α^* increases, v decreases in the range of $0 \le x \le 1.17$, but the opposite happens in the range of $1.17 \le x \le 10$, and converges to zero at $x \ge 10$. Figure 2 shows the distribution of temperature θ. It shows that in the range of $x \ge 0.29$, as the size of the parameter α^* increases, the temperature value decreases and converges to zero at $x \ge 10$. Figure 3 shows the distribution of the variation of the volume fraction field φ, where for all values of α^* it starts at negative values, starts at a minimum, increases in the range $0 \le x \le 10$ and converges to zero at $x \ge 10$. Figure 4 clarifies the distribution of the stress component σ_{xx} versus x. It is observed that the stress component σ_{xx} complies with the boundary conditions, starts from a negative value and reaches the minimum value in the range $0 \le x \le 1.2$, and increases in the range of $1.2 \le x \le 10$, and then converges to zero at $x \ge 10$. Figure 5 shows the distribution of the stress component σ_{xy} versus x. It has been observed that the linear temperature coefficient α^* has a strong influence on the distribution of σ_{xy}, as the value of α^* increases, the magnitude of σ_{xy}, decreases and all curves converge to zero.

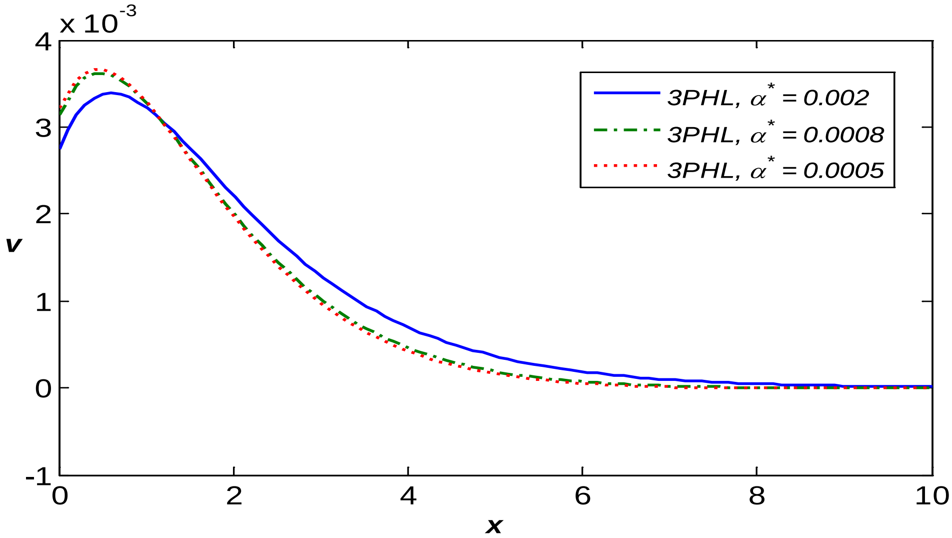

Fig. 1. Vertical displacement distribution v for different values of linear temperature coefficient.

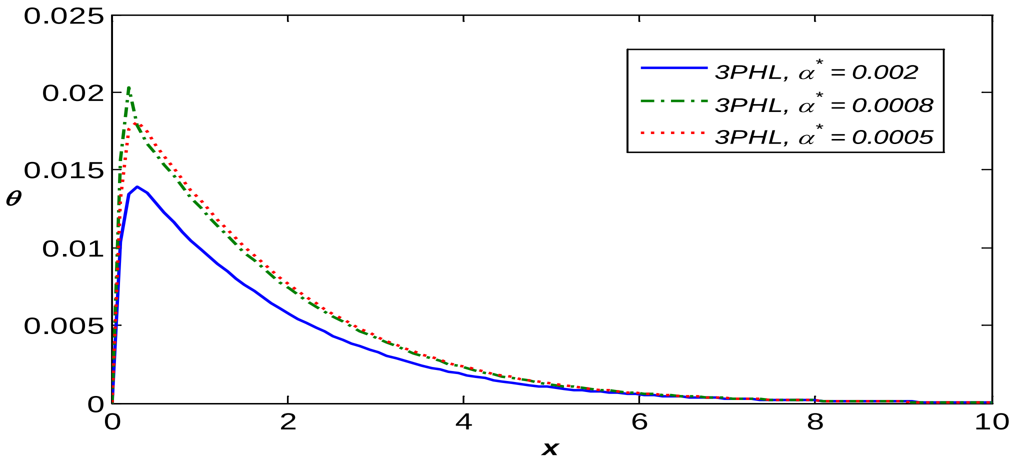

Fig. 2. Thermal temperature distribution θ for different values of linear temperature coefficient.

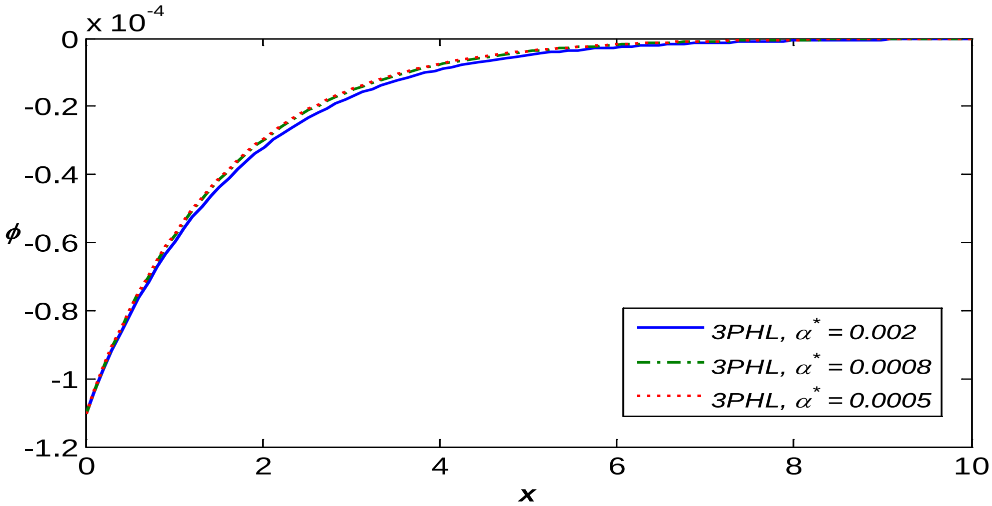

Fig. 3. The change in volume fraction field φ for different values of linear temperature coefficient.

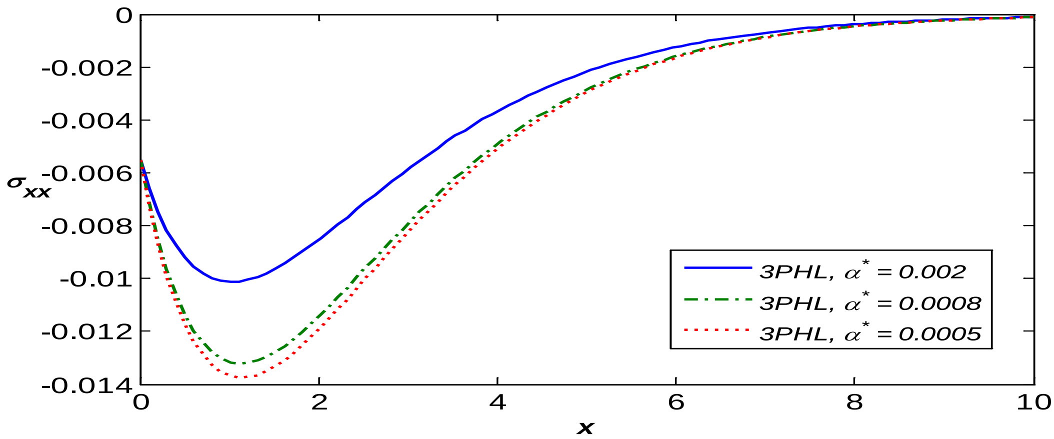

Fig. 4. Distribution of stress component σ_{xx} for different values of linear temperature coefficient.

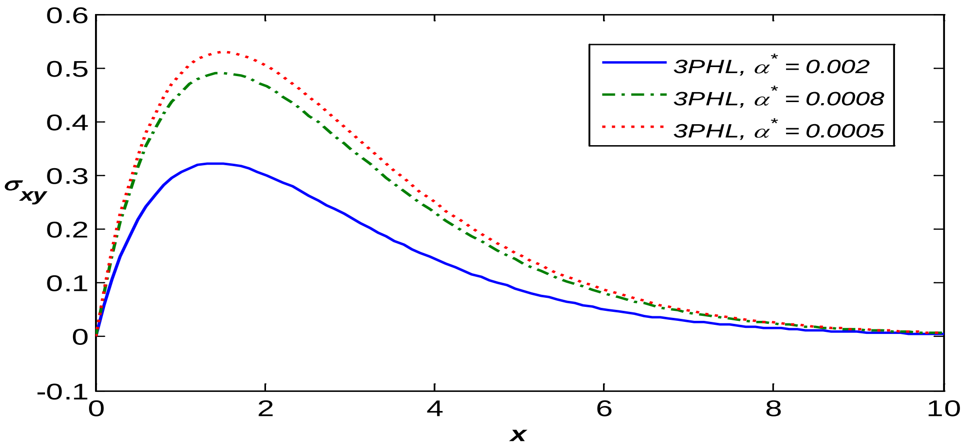

Fig. 5. Distribution of stress component σ_{xy} for different values of linear temperature coefficient.

Case II: Plot Figures 6-8 to show the effect of the nonlocal parameters on the variation of the displacement component v, thermal temperature θ and stress component σ_{xx} of the medium. In Figure 7, the effect of parameter ε on the variation of thermal temperature θ is the same as in Figure 2, but in the range of $x \geq 1.3$, the value of ε increases while the magnitude of θ decreases. In Fig. 8, the value of the stress component σ_{xx} increases when the parameter ε is also increased, it is clear that the distribution of the stress component σ_{xx} starts from a negative value, agrees with the boundary conditions and converges to zero $x \geq 10$.

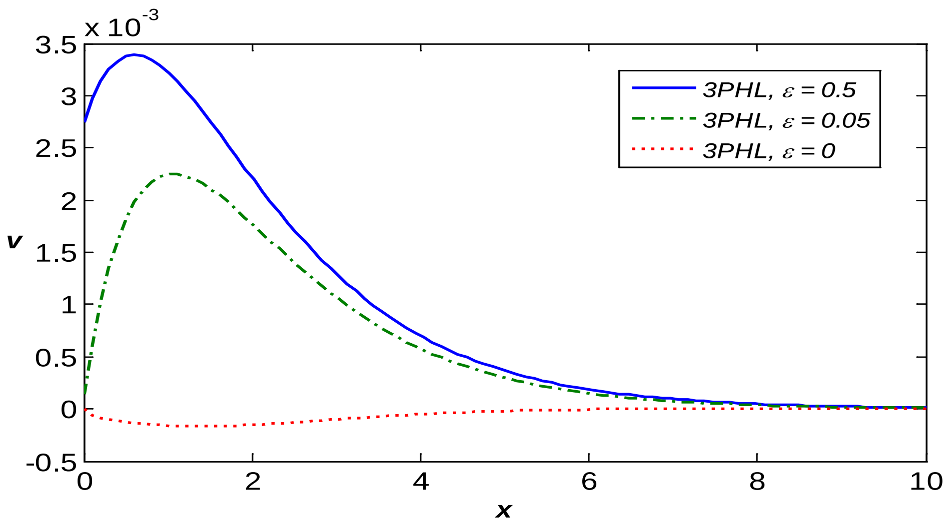

Fig. 6. Vertical displacement distribution v for different values of nonlocal parameter.

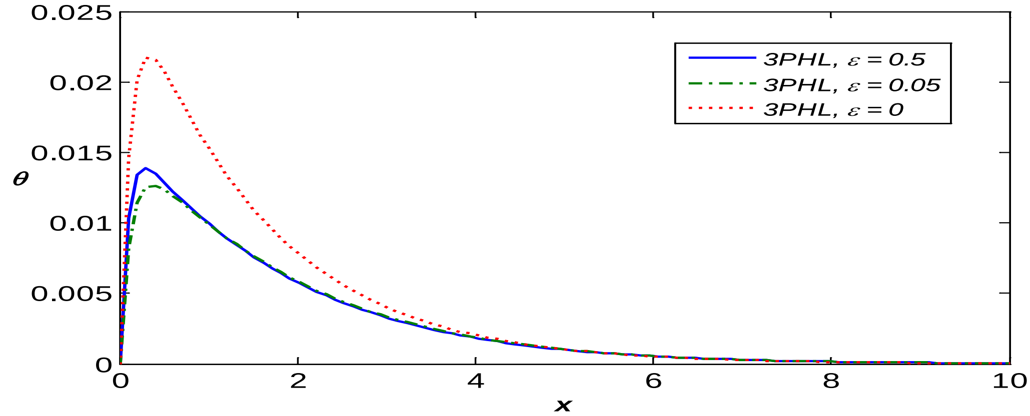

Fig. 7. Thermal temperature distribution θ for different values of nonlocal parameter.

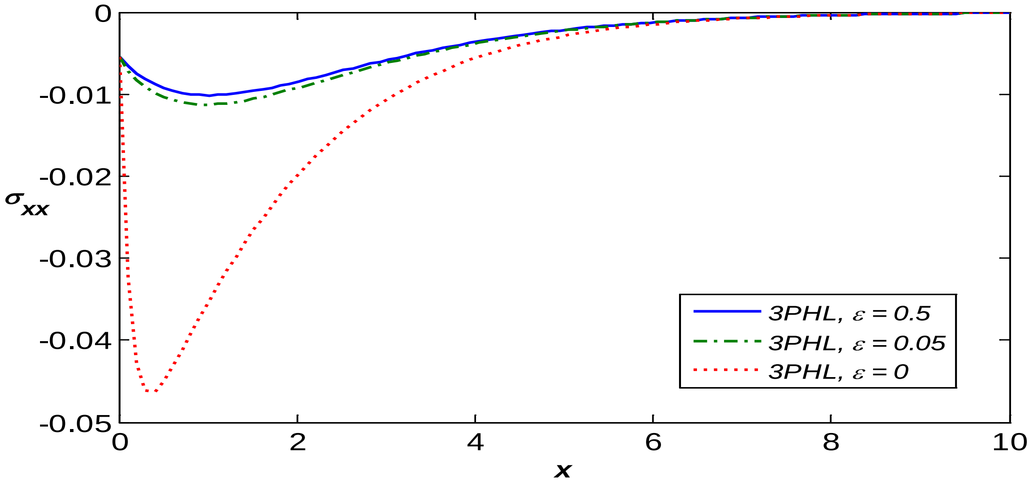

Fig. 8. Distribution of stress component σ_{xx} for different values of of nonlocal parameter.

Figures 9 and 10 are 3D curves showing the distribution of the volume fraction field φ and the stress component σ_{xy}associated with the three-phase delay model as a function of distance.

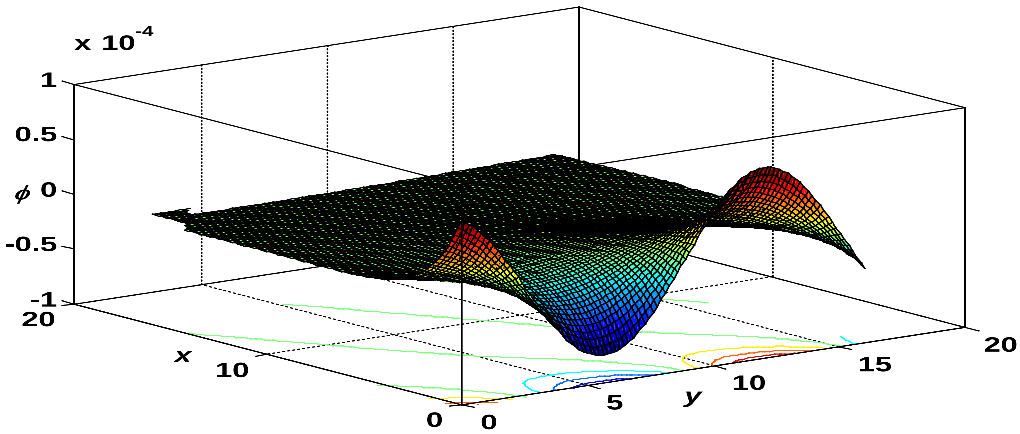

Fig. 9. The change in volume fraction field φ in the context of three-phase-lag model

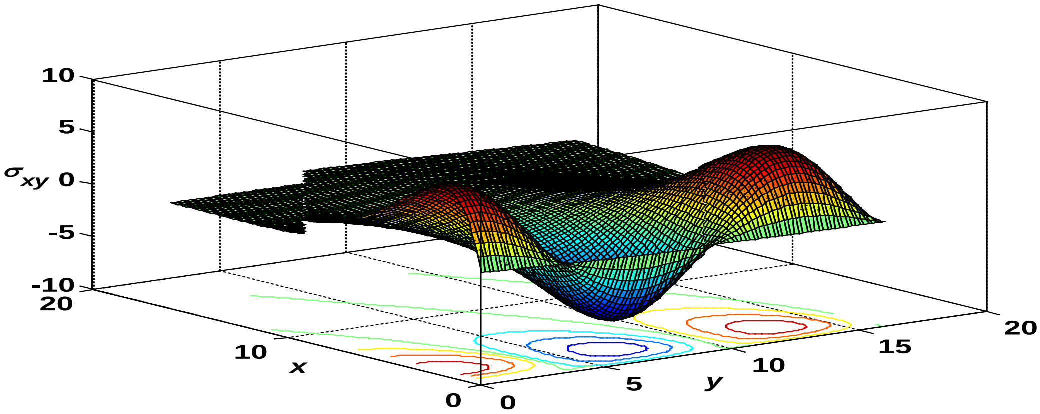

Fig. 10. Distribution of stress component σ_{xy} in the context of three-phase-lag model

6. Conclusion

In this study, we examine the effect of the linear temperature coefficient on a poro-thermoelastic medium. Here, the nonlocal model of thermoelastic generalizations is taken into account. The components of displacements and stresses were derived in view of the dimensionless parameters in the dynamical equations by utilizing the normal mode solution approach. Furthermore, the results are compared under the three-phase-lag model. The results of the above analysis can be summarized as follows:

1) The linear temperature coefficient plays an important role in the physical domain, as can be clearly seen from Figures 1-5.

2) From Figures 6-8, it can be seen that locality plays an important role in the physical fields.

3) Developed and used an analytical solution to the thermoelastic problem of solids based on the normal mode analysis.

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